

# In Search of Logical Bases beneath the Superficially Irrational

Dr. Kimihiko Yamagishi



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Dr. Kimihiko Yamagishi studies the way in which people incorrectly interpret statistical information. Here he discusses the concept of ratio bias and how it relates to a new psychophysical model he has developed which accurately predicts irrational behaviour when analysing statistical information.



**To begin, what is your research background and what attracted you to studying ratio bias?**

My interest in people's handling of statistical information dates back to high school. In high-school mathematics, my favorite subject was probabilities (I'll admit, I was an oddball). Learning probabilities and statistics made me aware that well-educated people casually make statistical mistakes. In one example, I was talking to an architect with a Ph.D. and a faculty position at a university about the test-taking tactics of students taking the TOEFL. He remarked, if a test-taker has no clue as to which is the correct answer out of 5 choices, fill out the bubble such that the test-taker's choices would make a zigzag line over the series of answer columns. His point was that this is the optimal answer tactic in the case of ignorance, because the correct answer columns in the TOEFL test are distributed randomly. I thought, "His point is not wrong, but inadequate. Assuming that the correct bubbles appear in the random positions, deciding upon any row, left-end, right-end, or middle, wherever, and filling all the bubbles in that row should suffice. His zigzag tactic requires some extra effort to make a zigzag line, involving wasteful cost." Thus as a high-school student, I made an early preparation for the study of statistical intuitions by observing that mistakes may occur.

**Your work unifies ratio bias literature by introducing a simple formula to explain a widely reported phenomenon. Considering you drew from other areas of study within psychology when formulating your theory, are there other areas of psychology to which it could be similarly applied? If so, what areas?**

Social psychology comes to my mind as the quickest example. When I presented this idea at a conference, I had a good conversation with a social psychologist who worked on self-esteem. We talked about how the kind of thinking which results in ratio bias would differ when applied to different types of personalities. Consider a person who took a standardized test and received a score in the 70th percentile. A high self-esteem individual would frame the result as "I am ahead of 70% of people" whereas a low self-esteem person would think, "I am worse off than 30% of people." Now what would happen if the total number of test takers were 100 or 10,000? A high self-esteem person would experience elation in the 10,000 case, knowing their score was better than 7,000 test takers rather than 70. In contrast, a low self-esteem person would experience depression in the 10,000 case, realizing that s/he is behind 3,000 competitors rather than merely 30.

**Your previous research suggested that base rate neglect resulted in ratio bias. What are the primary differences between base rate**

**neglect and psychophysical phenomenon? Are they mutually exclusive?**

I do not necessarily regard the "base rate neglect" and the psychophysical formulation as mutually exclusive. Rather, I would like to characterize the Dual Weber-Fechner theory as a process articulation of the "base-rate neglect" or "insensitivity to base rates." The literature uses the expressions in the quotation without describing the cognitive processes behind the phenomena. Thus I am happy to characterize that the common expressions, "base-rate neglect" or "insensitivity to base rates," as labels attached to the observable phenomena, and the Dual Weber-Fechner theory as a possible description of psychological processes that produce the observed phenomena.

**Clearly, ratio bias is an important concept for anyone working to educate the public. Are there industries you would like to see apply this knowledge?**

I would welcome positive contributions of the notion of ratio bias studies in risk education. Since the 2011 Tohoku Earthquake and Tsunami incident, Japanese researchers, politicians, activists and such are keenly aware of the relevance of risk education.

# The Power of Small Modifications

Ratio bias is an oft-cited but little understood psychological phenomenon. Dr. Kimihiko Yamagishi of the Tokyo Institute of Technology modifies a previously-existing psychophysical model and is now able to predict irrational statistical reasoning in cases of ratio bias.

## A COGNITIVE MYSTERY EXPLAINED

Logically inconsistent behavior in humans has always been recorded and studied by psychologists. Humans are clearly logical creatures; developmental psychology demonstrates that infants continually try to identify the logical relationships between every object they encounter. However, humans consistently use irrational decision-making processes, and the work of Dr. Kimihiko Yamagishi attempts to define when and how micro-logical thinking processes go awry in the context of the human logical system as a whole.



Understanding the way in which humans assess risk and make decisions has many implications in the psychological field. Ratio bias is frequently demonstrated in the literature, but little research had been conducted to develop a model that would explain the phenomenon. Yamagishi's ability to predict intuitive and irrational risk assessment calculations deepens the scientific community's understanding of the thought processes which result in ratio bias.

**I realized that tweaking a textbook psychological notion, the Weber-Fechner Law, could well explain the base-rate neglect phenomena after minor modifications.**

## UNDERSTANDING RATIO BIAS

Ratio bias has long perplexed psychological researchers. This phenomenon occurs when comparing ratios and statistical information. In an often-cited study conducted by Yamagishi in 1997, he presents two different experiments in order to demonstrate ratio bias. In the first experiment, he presented test subjects with eleven well known causes of death. Subjects were also presented with an estimation of yearly deaths for each cause in the form of

a ratio. The estimates were the result of a previous study in which test subjects were asked to guess the number of deaths due to each cause listed based on their intuition. There were four types of ratios: small numerators over a narrow range (denominator), large numbers over a narrow range, large numbers over a wide range, and small numbers over a wide range. Each participant viewed only one type of ratio, and the type of ratio received was randomized across test subjects. Participants were then asked to rate the riskiness of that particular cause of death on a scale of zero to 25 where zero represented no risk and 25 was the maximum amount of risk. The results indicated that larger numbers in the denominator, no matter what percentage is represented, are judged as riskier. Large and small numbers over wide ranges were consistently given higher ratings than small and large numbers over narrow ranges.

In the second experiment, the design was essentially the same as the first, but used only ratios with small numbers over a wide range and large numbers over a narrow range. This design was used so that subjects were only presented with small percentages represented with large numbers or large percentages represented with small numbers. For example, one group of individuals were asked to assess

the phrase “cancer kills 1,286 out of 10,000 people” while other participants assessed the phrase “cancer kills 24.14 out of 100”. The first statement describes a percentage of 12.86% while the second statement denotes a 24.14% chance of developing cancer. The results corroborated the results of the first experiment; test subjects rated the first phrase representing a 12.86% chance as riskier than the second phrase representing a 24.14% chance.

At the time, Yamagishi attributed this inability to accurately compare ratios as a case of “base-rate neglect”, or a tendency to emphasize certain information and ignore equally important facts when making judgments. For example, in this experiment, subjects only consider the magnitudes of the numerators and ignore its relation to the magnitude of the denominators. While base-rate neglect is still relevant to the study of ratio bias, in 2007, Yamagishi was able to develop a new model to accurately predict ratio bias. This model describes the psychological process used in decision making.

#### A NEW PSYCHOPHYSICAL MODEL

The Weber-Fechner Law, a classic principle in psychology, states that changes perceived in subjectively-experienced stimuli are proportional to the stimulus magnitude. For instance, the addition of one-kilogram to a person holding a five-kilogram weight will feel the same to a person who is holding a ten-kilogram weight and adds two kilograms. The Weber-Fechner Law also applies to changes in noise, light, or any time human perception inaccurately calculates the magnitude of a stimulus. Mathematically, the calculation consists of taking the logarithm of the ratio of the perceived intensity over the actual, measured intensity of the phenomena.

The Dual Weber-Fechner Theory developed by Yamagishi modifies the formula to calculate the psychological magnitude separately for both the numerator and the denominator. With this new model, he accurately predicted the behavior of the study subjects in his work produced ten years before in 1997. The Weber-Fechner Law, applied to Yamagishi’s previous research, would be calculated as  $\log(1,286/10,000)$ , which results in an answer of 0.8, and  $\log(24.14/100)$ , which results in an answer of 0.9. A larger number denotes the ratio most psychologically significant in the human mind, so the Weber-Fechner Law does

not account for this irrational decision-making. Now, with the Dual Weber-Fechner Theory, the calculations become  $\log(1,286)/\log(10,000)$  and  $\log(24.14)/\log(100)$ . These calculations give answers of 0.78 and 0.64, respectively, and now account for the odd behavior displayed by test subjects.

#### TESTING THE THEORY

When asked if a meta-analysis across ratio bias literature would be an appropriate way to test his theory, Yamagishi says: “Having to test all the existing cases of ratio bias phenomena seems like a form of “Probatio diabolica,” or asking for the virtually impossible”, as the cases of ratio bias in the literature are so extensive. However, the Dual Weber-Fechner theory accurately describes the outcomes of key studies and influential research conducted by others. Yamagishi applied the Dual Weber-Fechner theory to a study published in 1989 by researchers Miller, McFarland and Turnbull in which participants were asked to rate the chances of winning a game in which the odds were either 2/20 or 20/200. While these ratios have the same probability, analysis by the Dual Weber-Fechner theory results in the calculations  $\log(2)/\log(20)$  and  $\log(20)/\log(200)$ . These calculations give answers of 0.167 and 0.565, respectively, explaining how participants would find odds of 20/200 more attractive.

In another well-known demonstration conducted in 1994 by Denes-Raj and Epstein, test subjects were offered a choice between the odds “1 in 10” and “9 out of 100”, where the odds represented the chances of winning a cash award. Participants irrationally chose the phrase “9 out of 100” as being more likely to result in winning the money than the phrase “1 in 10”. The Dual Weber-Fechner theory explains this result as well. In this case, the psychophysical function  $\log(Q+0.01)$  must be applied in order to account for the fact that  $\log(1)=0$ , and would thus strip the calculation of its usefulness. The odds “1 in 10” would therefore be calculated as  $\log(1.01)/\log(10.01)$ , resulting in an answer of 0.004. The odds “9 in 100” are calculated as  $\log(9.01)/\log(100.01)$  and give an answer of 0.477. The Dual Weber-Fechner theory, again, accounts for the irrational choice of preferring a 9% chance of winning money to a 10% chance. Yamagishi’s work and its ability to explain multiple cases of ratio bias in the literature demonstrates the amazing explanatory power of this simple model.

## Researcher Profile



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Dr Kimihiko Yamagishi received his Ph.D. from the University of Washington. He has been recognized by the Japan Psychological Association with the Distinguished Paper Award in 2003 and 2006, and received the Special Award at the 2012 Annual Conference of the Japan Cognitive Science Society. Dr Yamagishi aims to understand how humans make decisions and when and how human perception does not reflect reality. He has published papers in the areas of risk perception, decision making, and probability judgement, and primarily uses controlled experimentation, surveys, and statistical and mathematical modelling to conduct his research.

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